Problem 12-2

If a particle has an initial velocity of $v_0 = 12$ ft/s to the right, at $s_0 = 0$, determine its position when $t = 10$ s, if $a = 2$ ft/s^2 to the left.

Solution

The fundamental relationship between acceleration $a = a(t)$ and velocity $v = v(t)$ is

$$a = \frac{dv}{dt}.$$

Substitute the given formula for the acceleration.

$$\frac{dv}{dt} = -2$$

Integrate both sides with respect to t.

$$v(t) = -2t + C_1$$

Use the given initial velocity to determine C_1.

$$v(0) = -2(0) + C_1 = 12 \quad \rightarrow \quad C_1 = 12$$

The particle's velocity (in feet per second) is then

$$v(t) = -2t + 12. \tag{1}$$

The fundamental relationship between velocity $v = v(t)$ and position $s = s(t)$ is

$$v = \frac{ds}{dt}.$$

Plug it into equation (1).

$$\frac{ds}{dt} = -2t + 12$$

Integrate both sides with respect to t.

$$s(t) = -t^2 + 12t + C_2$$

Use the given initial position to determine C_2.

$$s(0) = -0^2 + 12(0) + C_2 = 0 \quad \rightarrow \quad C_2 = 0$$

The particle's position (in feet) is then

$$s(t) = -t^2 + 12t.$$

Therefore, at $t = 10$ s the position is

$$s(10) = -10^2 + 12(10) = 20 \text{ ft}.$$